Sample Question Paper - 4
Mathematics (041)
Class- XII, Session: 2021-22
TERM II

Time Allowed: 2 hours   **Maximum Marks: 40**

General Instructions:

1. This question paper contains three sections – A, B and C. Each part is compulsory.

2. Section - A has 6 short answer type (SA1) questions of 2 marks each.

3. Section – B has 4 short answer type (SA2) questions of 3 marks each.

4. Section - C has 4 long answer-type questions (LA) of 4 marks each.

5. There is an internal choice in some of the questions.

6. Q 14 is a case-based problem having 2 sub-parts of 2 marks each.

Section A

1. Evaluate: $\int \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)dx$ **[2]**

OR

Evaluate: $\int \frac{\sec^2 \sqrt{x}}{\sqrt{x}}dx$

2. Solve the initial value problem: $(xe^{y/x} + y)\, dx = x\, dy$, $y(1) = 1$ **[2]**

3. For what value of λ are the vectors $\vec{a}$ and $\vec{b}$ perpendicular to each other? Where   **[2]**
   $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{j} - \lambda\hat{k}$

4. Find the direction cosines of the line $\frac{x-2}{2} = \frac{2y-5}{-3}$, z = -1. Also, find the vector equation of the   **[2]**
   line.

5. An experiment succeeds twice as often as it fails. Find the probability that in the next six   **[2]**
   trails, there will be at least 4 successes.

6. A bag contains 4 red and 5 black balls, a second bag contains 3 red and 7 black balls. One ball   **[2]**
   is drawn at random from each bag, find the probability that the balls are of the same colour.

Section B

7. Evaluate $\int \frac{2x+1}{\sqrt{3x+2}}dx$ **[3]**

8. Solve the following differential equation.   **[3]**
   $\cos^2 x \frac{dy}{dx} + y = \tan x$

OR

Form the differential equation of the family of circles touching the y - axis at the origin.

9. For any two vectors $\vec{a}$ and $\vec{b}$ prove that: $|\vec{a} + \vec{b}|^2 + |\vec{a} - \vec{b}|^2 = 2\left(|\vec{a}|^2 + |\vec{b}|^2\right)$.   **[3]**

10. A line makes angles α, β, γ and δ with the diagonals of a cube, prove that   **[3]**
    $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2 \delta = \frac{4}{3}$

OR

Find the shortest distance between the lines whose vector equations are
$$\vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k}) \text{ and } \vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k}).$$

Section C

11. Evaluate: $\int \frac{dx}{\sin x(3+2\cos x)}$. **[4]**

12. Draw a rough sketch of the region $f(x,y) : y^2 \leq 5x, 5x^2 + 5y^2 \leq 36\}$ and find the area **[4]** enclosed by the region using method of integration.

OR

Using integration, find the area of the region enclosed between the two circles x² + y² = 4 and (x - 2)² + y² = 4.

13. Find the shortest distance between the given lines. $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k}),$ **[4]** $\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(-2\hat{i} + 3\hat{j} + 8\hat{k})$

CASE-BASED/DATA-BASED

14. In an office three employees Govind, Priyanka and Tahseen process incoming copies of a **[4]** certain form. Govind process 50% of the forms, Priyanka processes 20% and Tahseen the remaining 30% of the forms. Govind has an error rate of 0.06, Priyanka has an error rate of 0.04 and Tahseen has an error rate of 0.03.

Based on the above information, answer the following questions.

  i. The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, the probability that the form is NOT processed by Govind is

 ii. Let A be the event of committing an error in processing the form and let E_1, E_2 and E_3 be the events that Govind, Priyanka and Tahseen processed the form. The value of $\sum_{i=1}^{3} P(E_i \mid A)$?

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